

# Mean-field reduction of a layered macroscopic network model

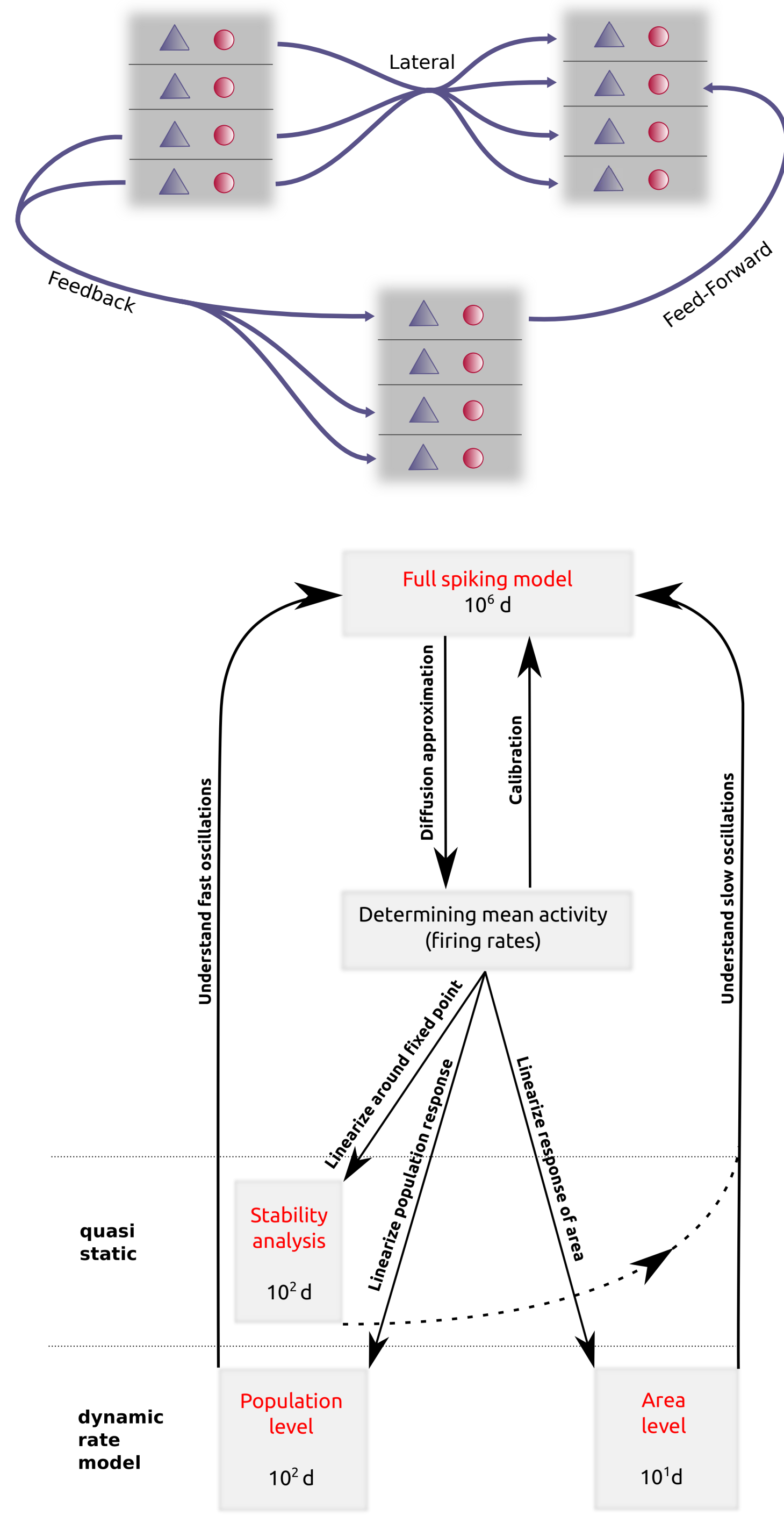
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## Motivation

- Understand mean activity and oscillations in a layered macroscopic model of the macaque visual cortex.  
**Visit poster P-1: Integrating multi-scale data for a network model of macaque visual cortex**
- Model details:
  - 32 visual areas modeled as single micro-columns
  - 254 neuron populations
  - currently  $6 \cdot 10^6$  neurons,  $1.2 \cdot 10^7$  dynamic variables
- Goal: Understand the dynamics of this high dimensional system**
- Method: Reduction of complexity**
  - coarser spatial description on the level of populations/areas
  - reduce temporal resolution, quasi static description



## Mean population activity

- Neuron dynamics

$$\tau_m \frac{dV}{dt} = -V + I_s(t)$$

$$\tau_s \frac{dI_s}{dt} = -I_s + \tau_m \sum_{i=1}^N J_i \delta(t - t_i - d)$$

- Stochastic description of the neuronal input current:

$$\tau_s \frac{dI_s}{dt} = -I_s + \mu + \sigma \sqrt{\tau_m} \eta(t)$$

- Self-consistent equation for stationary firing rates [2]:

$$\frac{1}{\nu_i} = \tau_r + \tau_m \sqrt{\pi} \int \frac{\frac{\mu_i - \mu_j}{\sigma_i} + \frac{\alpha}{2} \sqrt{\frac{\tau_s}{\tau_m}}}{\frac{\nu_j - \mu_j}{\sigma_j} + \frac{\alpha}{2} \sqrt{\frac{\tau_s}{\tau_m}}} e^{s^2} (1 + \text{erf}(s)) ds$$

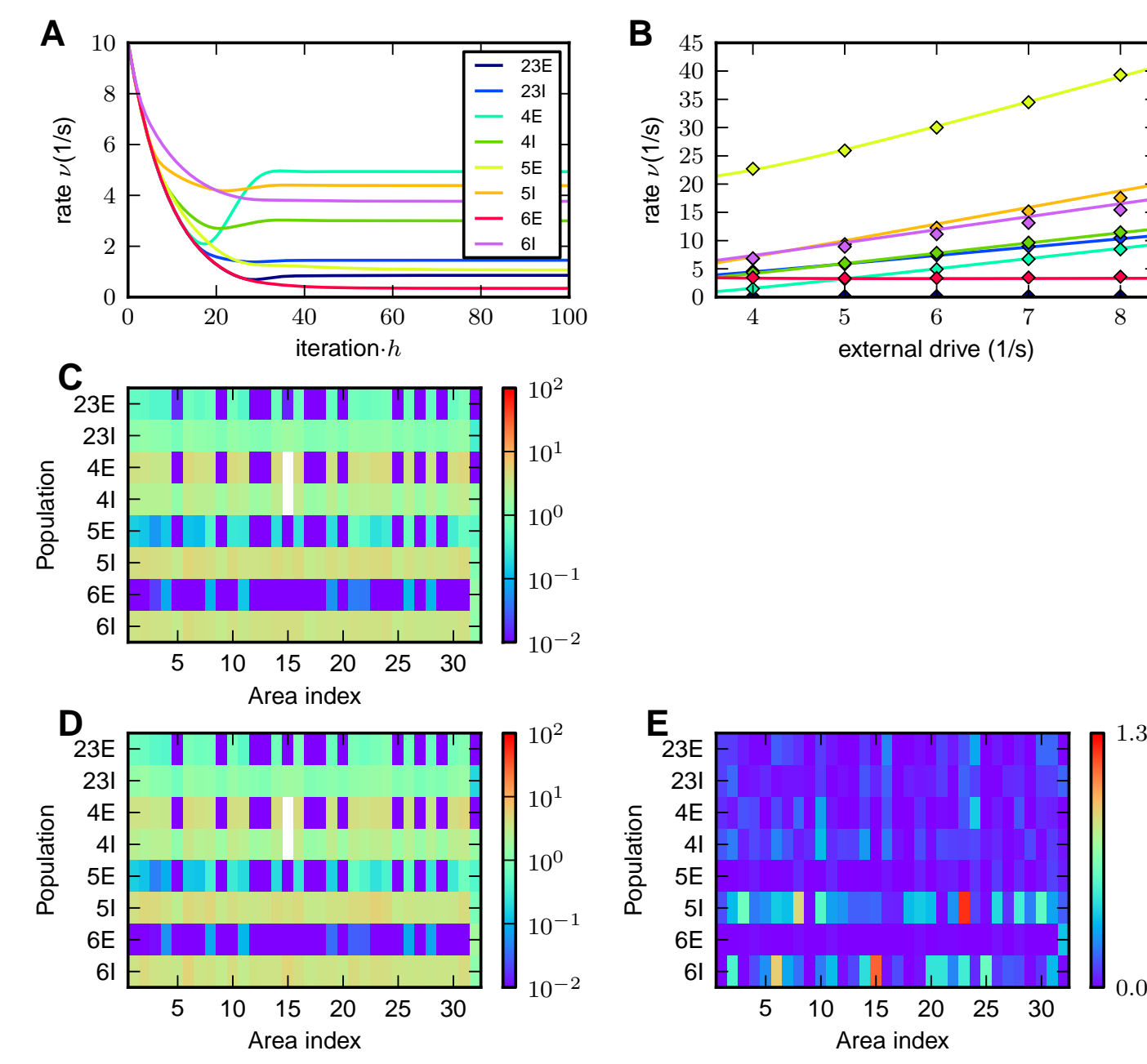
$$\equiv \frac{1}{\Phi_i(\nu)}$$

$$\mu_i = \sum_j \tau_m (KJ)_{ij} \nu_j$$

$$\sigma_i = \sum_j \tau_m (KJ^2)_{ij} \nu_j$$

- Finding the fixed point by integrating first order quasi dynamics [6]:

$$\frac{d\nu}{dt} = \Phi(\nu) - \nu \quad (1)$$

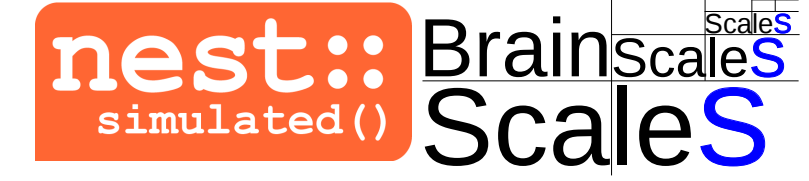
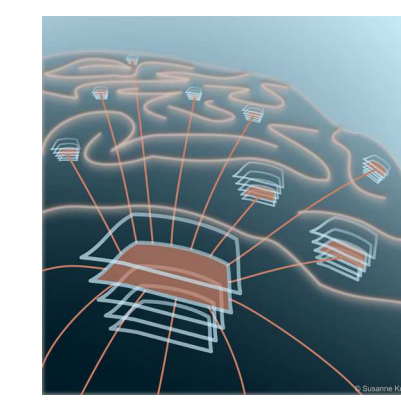


A Integration of (1) with timestep  $h$   
 B Firing rates in different layers of isolated area V1 depending on the external drive  
 C Theoretical firing rates (fixed point solution) in all 32 areas  
 D Simulated firing rates corresponding to C  
 E Deviation  $\Delta\nu$  between theory (C) and simulation (D)

## References

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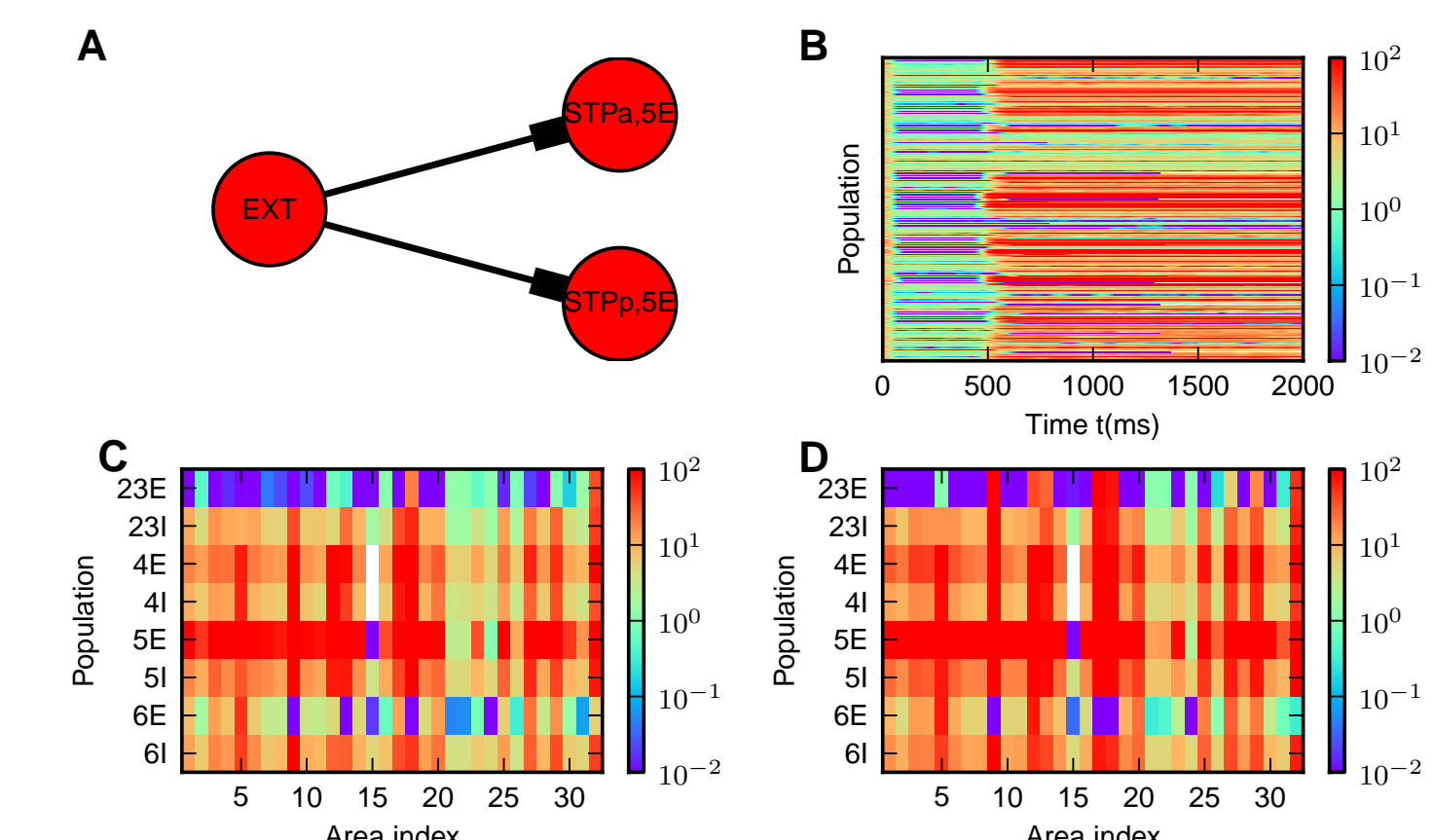


## Stability analysis

- Main model parameters: external drive and ratio  $g$  of inhibitory to excitatory weights
- Bistable system: stable low and high activity state for a large range of  $g$
- Time Evolution of a small perturbation  $\epsilon$  at  $t = t_p$ :

$$\frac{d\epsilon}{dt} = \underbrace{(-1 + \Phi'W)}_J \epsilon$$

- Exponential growth if  $\text{Re}(\lambda_j) > 0$
- During transient phase the system is sensitive to a perturbation in the direction of the populations STPa,5E and STPp,5E
- External excitation of STPa,5E and STPp,5E in the spiking Simulation leads to a transition from the low to the high activity state
- In general the network can be stabilized by modifying the number of II-synapses in the layer 5 E-I subnetworks.
- After stabilization and increase of external input to excitatory populations, the network shows slow oscillating activity



A Illustration of the external excitation of the spiking network  
 B Time evolution of firing rates encoded in color. External excitation applied at  $t = 500ms$  for a duration of  $300ms$

- C Mean-field solution for high activity state  
 D Simulated mean firing rates after transition to the high activity state

## Dynamical mean-field description

- Spiking model can be mapped to a linear rate model [3,4]

$$\mathbf{r}(t) = \mathbf{h}(\circ) * [\mathbf{w}(\mathbf{r}(\circ - d) + \mathbf{x}(\circ))](t) \quad (2)$$

- Impulse response  $\mathbf{h}(\circ)$  was derived analytically in frequency domain, i.e.  $\tilde{H}(\omega)$  for  $\delta$ -shaped currents

- Back transform via residue theorem reveals important time constants [5]

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} \tilde{H}(\omega) d\omega$$

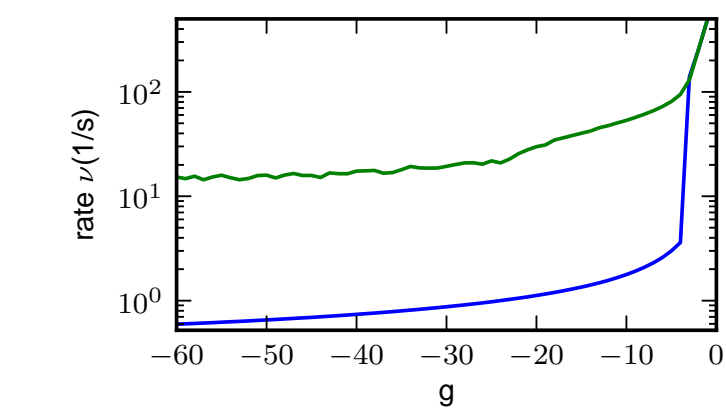
$$= \sum_{\omega_0} \text{Res}(\tilde{H}, \omega_0) e^{i\omega_0 t} \Theta(t)$$

- Population power- and cross spectra can be derived from (2)

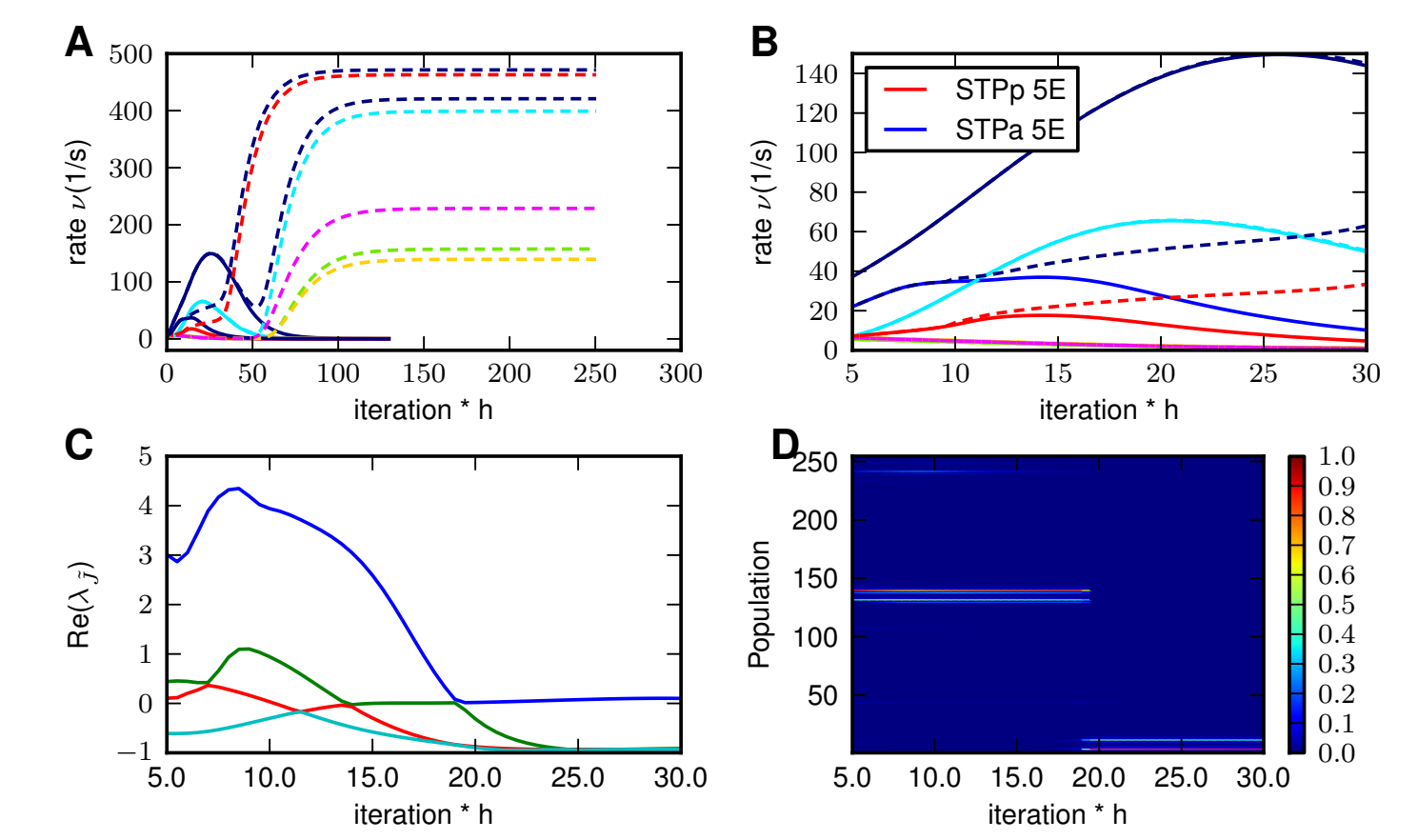
$$\mathbf{C}(\omega) = \mathbf{P}(\omega) \mathbf{D} \mathbf{P}^T(\omega) \quad (3)$$

$$\mathbf{P}(\omega) = (\mathbf{H}_d^{-1}(\omega) - \mathbf{W})^{-1} \mathbf{W} + 1 \quad (4)$$

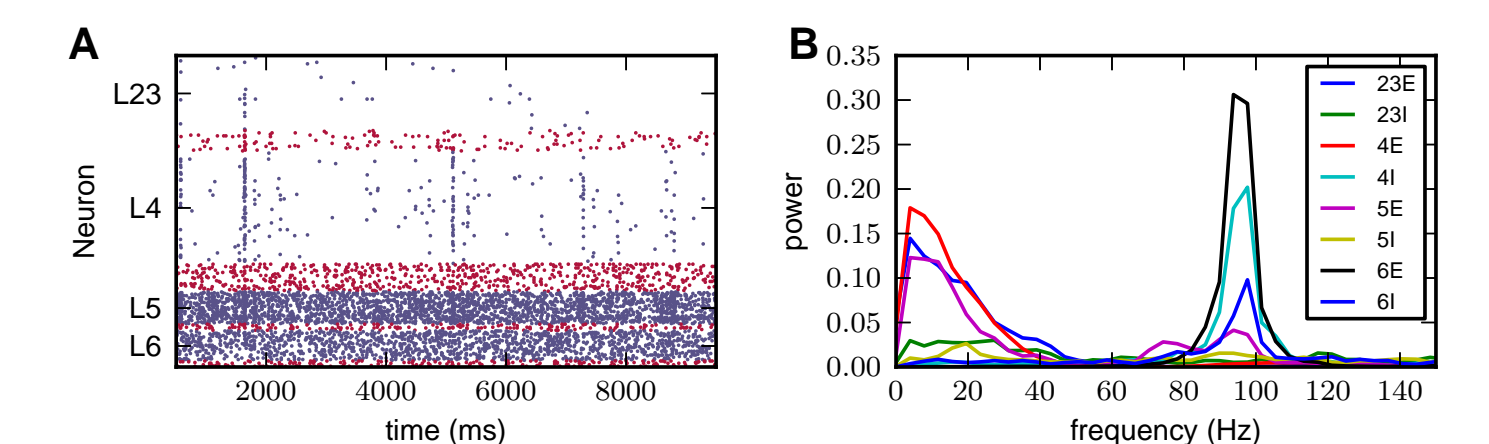
- Significant peaks in power spectra of populations can be explained by considering only the most important time constant



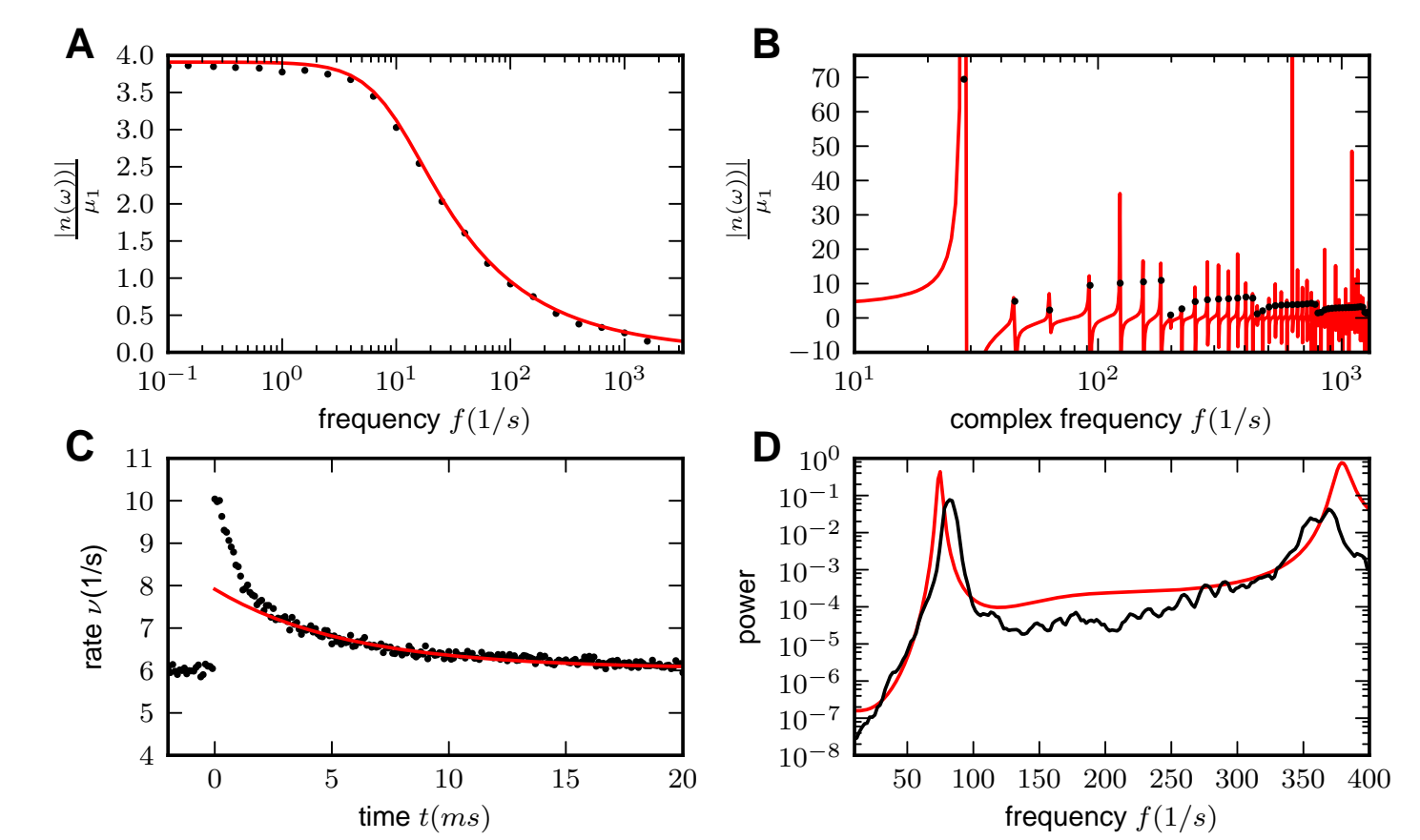
Hysteresis curve for a variation of parameter  $g$ .  
 Blue line: Increase of  $g$ . Green line: Decrease of  $g$ .



A Solid line: Trajectory of firing rates for eight different populations, representative for all 254 populations. Dashed line: perturbed at iteration  $\cdot h = 10$  in direction of most unstable eigenvector  
 B Zoom of transient phase in A  
 C Four largest eigenvalues of  $\tilde{J}$  during transient phase  
 D Time evolution of 254 dimensional eigenvector, corresponding to largest eigenvalue during transient phase. Absolute value of each component encoded in color.



A Raster plot for area V1, embedded in the whole network model  
 B Firing rate spectra for the eight different populations in V1, corresponding to A



A Red line: Transfer function in frequency domain. Black Dots: Simulation.  
 B Red line: Analytical continuation of transfer function over purely imaginary frequency axis. Black dots: value of  $\text{Res}(\tilde{H}, \omega_0)$  at purely imaginary poles  $\omega_0$  (same  $y$ -axis).  
 C Black dots: Firing rate response from simulation (dots) approximated by largest residue and corresponding pole/time constant.  
 D Black line: Power spectrum for population 4l in isolated area 'V1'. Black line: Simulation. Red line: Analytical prediction (3) using transfer function approximated as in C.

## Outlook

- Slow oscillations can emerge in spiking neural networks in the meta-stable regime due to rate instabilities [1]
- Identify mechanisms and recurrent loops controlling fast and slow oscillations via dynamical mean-field description and stability analysis
- Study emergence of slow oscillations by reducing single areas to input-output relationships (area transfer function)
- Extraction of macroscopic signal (BOLD) from the model for comparison to empirically observed slow oscillations, i.e. resting state activity
- Comparison of functional connectivity maps between model and experiment